Exercise 1.2

nth Root:

If **n** is a positive integer greater than **1** and **a** & **x** are real numbers such that $x = \sqrt[n]{a}$. Then **x** is called n*th* root of **a**.

Note:

- $\sqrt{\square}$ is called radical
- **n** is called index of radical
- **a** is called radicand

Examples:

$\sqrt[3]{5}$ and $\sqrt[5]{7}$ are examples of radical form.

Surd:

An irrational radical with rational radicand is called a surd.

Examples:

- $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}, \sqrt{3}+\sqrt{5}$ are surds
- $\sqrt{\pi}, \sqrt{e}$ are not surds

Types of Surds

(1) **Monomial surd:**

A surd that contains a single term is called monomial surd.

Example: $\sqrt{5}, \sqrt{7}$ etc.

(2) **Binomial surd:**

A surd that contains two monomial surds is called binomial surd.

Example: $\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{7}$ etc.

(3) **Conjugate surds:**

$\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

Example:

$\sqrt{3}+\sqrt{5}$ & $\sqrt{3}-\sqrt{5}$ are conjugate surds of each other.

Rationalization of Denominator:

To rationalize a denominator of the form $\sqrt{a}+\sqrt{b}$ or $\sqrt{a}-\sqrt{b}$, we multiply numerator and denominator by conjugate factor.

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Solution of Exercise

Question No.1: Rationalize the denominator of the following:

(i) $\frac{13}{4+\sqrt{3}}$

Solution:

$$\frac{13}{4+\sqrt{3}}=\frac{13}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{\left(4+\sqrt{3}\right)\left(4-\sqrt{\mathbf{3}}\right)}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{(4)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{16-3}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{13}$$

$$=\mathbf{4-\sqrt{3}}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Solution:

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}=\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{3}\left(\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{3}\right)^2}$$

$$=\frac{\sqrt{\mathbf{6}}+\sqrt{\mathbf{15}}}{\mathbf{3}}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution:

$$\frac{\sqrt{2}-1}{\sqrt{5}}=\frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$=\frac{\sqrt{5}\left(\sqrt{2}-1\right)}{\left(\sqrt{5}\right)^2}$$

$$=\frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Solution:

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}=\frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$=\frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$$

$$=\frac{\left[2\left(3-2\sqrt{2}\right)\right]^2}{36-16\times 2}$$

$$=\frac{4\left(3-2\sqrt{2}\right)^2}{36-32}$$

$$=\frac{4\left(3-2\sqrt{2}\right)^2}{4}$$

$$=\left(3-2\sqrt{2}\right)^2$$

$$=9-2\times 3\times 2\sqrt{2}+\left(2\sqrt{2}\right)^2$$

$$=9-12\sqrt{2}+4\times 2$$

$$=\mathbf{17-12\sqrt{2}}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Solution:

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{3-2}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{1}$$

$$= \left(\sqrt{3} - \sqrt{2}\right)^2$$

$$= \left(\sqrt{3}\right)^2 - 2\times \sqrt{3} \times \sqrt{2} + \left(\sqrt{2}\right)^2$$

$$= 3 - 2\sqrt{6} + 2$$

$$\mathbf{= 5 - 2\sqrt{6}}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solution:

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} = \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$$

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{2}$$

$$\mathbf{= 2\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}$$

Question No.2: Simplify the following:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Solution:

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$$

$$= \left[\frac{(2)^4}{(3)^4}\right]^{\frac{3}{4}}$$

$$= \frac{(2)^{4\times\frac{3}{4}}}{(3)^{4\times\frac{3}{4}}}$$

$$= \frac{(2)^3}{(3)^3}$$

$$= \mathbf{\frac{8}{27}}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

Solution:

$$\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$$

$$= \left(\frac{4}{3}\right)^{2} \times \left(\frac{9}{4}\right)^{3} \times \frac{16}{27}$$

$$= \frac{4^2}{3^2} \times \frac{9^3}{4^3} \times \frac{16}{27}$$

$$= \frac{4^2}{9} \times \frac{9\times9\times9}{4^2\times4} \times \frac{16}{9\times3}$$

$$= \frac{4^2}{9} \times \frac{9\times9\times9}{4^2\times4} \times \frac{16}{9\times3}$$

$$= \frac{1}{9} \times \frac{9\times9\times9}{1\times4} \times \frac{16}{9\times3}$$

$$= \frac{1}{1} \times \frac{1\times9\times9}{4} \times \frac{16}{9\times3}$$

$$= \frac{9\times 9}{1} \times \frac{4}{9\times 3}$$

$$= \frac{1\times 9}{1} \times \frac{4}{1\times 3}$$

$$= \frac{9\times 4}{3}$$

$$= \frac{3\times 4}{1}$$

$$= \mathbf{12}$$

(iii) $(0.027)^{-\frac{1}{3}}$

Solution:

$$(0.027)^{-\frac{1}{3}} = \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$

$$= \left(\frac{1000}{27}\right)^{\frac{1}{3}}$$

$$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$

$$= \frac{(10)^{3\times\frac{1}{3}}}{(3)^{3\times\frac{1}{3}}}$$

$$= \mathbf{\frac{10}{3}}$$

(iv) $\sqrt[7]{\frac{x^{14}\times y^{21}\times 3^{35}}{y^{14}z^{7}}}$

Solution:

$$\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$$

$$= \left(\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}\right)^{\frac{1}{7}}$$

$$= \frac{(x)^{14\times\frac{1}{7}}\times(y)^{21\times\frac{1}{7}}\times(z)^{35\times\frac{1}{7}}}{(y)^{14\times\frac{1}{7}}\times(z)^{7\times\frac{1}{7}}}$$

$$= \frac{x^2\times y^3\times z^5}{y^2\times z}$$

$$= x^2 \times y^{3-2} \times z^{5-1}$$

$$= \boldsymbol{x^2yz^4}$$

(v) $\frac{5\cdot(25)^{n+1}-25\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-(25)^{n+1}}$

Solution:

$$\frac{5\cdot(25)^{n+1}-25\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-(25)^{n+1}}$$

$$= \frac{5\cdot\left(5^2\right)^{n+1}-25\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-\left(5^2\right)^{n+1}}$$

$$= \frac{5\cdot(5)^{2(n+1)}-25\cdot(5)^{2n}}{5\cdot(5)^{2n}\cdot(5)^3-(5)^{2(n+1)}}$$

$$= \frac{5\cdot(5)^{2n+2}-25\cdot(5)^{2n}}{5\cdot(5)^{2n}\cdot125-(5)^{2n+2}}$$

$$= \frac{5\cdot(5)^{2n}\cdot5^2-25\cdot(5)^{2n}}{5\cdot(5)^{2n}\cdot125-(5)^{2n}\cdot(5)^2}$$

$$= \frac{5\cdot(5)^{2n}\cdot25-25\cdot(5)^{2n}}{5\cdot(5)^{2n}\cdot125-(5)^{2n}\cdot25}$$

$$= \frac{(5)^{2n}\cdot125-25\cdot(5)^{2n}}{(5)^{2n}\cdot625-(5)^{2n}\cdot25}$$

$$= \frac{(5)^{2n}[125-25]}{(5)^{2n}[625-25]}$$

$$= \frac{125-25}{625-25}$$

$$= \frac{100}{600}$$

$$= \boldsymbol{\frac{1}{6}}$$

(vi) $\frac{(16)^{x+1}+20\left(4^{2x}\right)}{2^{x-3}\times 8^{x+2}}$

Solution:

$$\frac{(16)^{x+1}+20\left(4^{2x}\right)}{2^{x-3}\times 8^{x+2}}$$

$$=\frac{\left[(2)^{4}\right]^{x+1}+20\left[(2)^{2}\right]^{2x}}{(2)^{x-3}\times\left[(2)^{3}\right]^{x+2}}$$

$$=\frac{(2)^{4(x+1)}+20(2)^{4x}}{(2)^{x-3}\times(2)^{3(x+2)}}$$

$$=\frac{(2)^{4x+4}+20(2)^{4x}}{(2)^{x-3}\times(2)^{3x+6}}$$

$$=\frac{(2)^{4x}\cdot(2)^{4}+20(2)^{4x}}{(2)^{x-3+3x+6}}$$

$$=\frac{(2)^{4x}\cdot 16+20(2)^{4x}}{(2)^{4x+3}}$$

$$=\frac{(2)^{4x}[16+20]}{(2)^{4x}\cdot(2)^{3}}$$

$$=\frac{16+20}{(2)^{3}}$$

$$=\frac{36}{8}$$

$$=\frac{\mathbf{9}}{\mathbf{2}}$$

(vii) $(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}}$

Solution:

$$(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}}$$

$$=\left[(4)^{3}\right]^{-\frac{2}{3}}\div\left[(3)^{2}\right]^{-\frac{3}{2}}$$

$$=\frac{\left[(4)^{3}\right]^{-\frac{2}{3}}}{\left[(3)^{2}\right]^{-\frac{3}{2}}}$$

$$= \frac{[(3)^2]^{\frac{3}{2}}}{[(4)^3]^{\frac{2}{3}}}$$

$$= \frac{(3)^{2\times\frac{3}{2}}}{(4)^{3\times\frac{2}{3}}}$$

$$= \frac{(3)^3}{(4)^2}$$

$$= \mathbf{\frac{27}{16}}$$

(viii) $\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Solution:

$$\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

$$=(3)^{n-(n-1)} \times (9)^{n+1-(n-1)}$$

$$= (3)^{n-n+1} \times (9)^{n+1-n+1}$$

$$= (3)^1 \times (9)^2$$

$$= 3 \times 81$$

$$= \mathbf{243}$$

(ix) $\dfrac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 4 \times 5^n}$

Solution:

$$\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 4 \times 5^n}$$

$$= \frac{(5)^n \cdot (5)^3 - 6 \cdot (5)^n \cdot (5)^1}{(5)^n [9-4]}$$

$$= \frac{(5)^n \cdot 125 - 6 \cdot (5)^n \cdot 5}{(5)^n \cdot 5}$$

$$= \frac{(5)^n [125 - 6\times 5]}{(5)^n \cdot 5}$$

$= \frac{125-30}{5}$

$= \frac{95}{5}$

$= \mathbf{19}$

Question No.3: If $x = 3 + \sqrt{8}$ *then find the value of*:

(i) $x + \frac{1}{x} = ?$

Solution:

$x = 3 + \sqrt{8} \qquad \ldots (1)$

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$$

$= \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$

$= \frac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}$

$= \frac{3-\sqrt{8}}{9-8}$

$= \frac{3-\sqrt{8}}{1}$

$\mathbf{\frac{1}{x} = 3 - \sqrt{8}} \qquad \ldots (2)$

Add eq. (1) and (2):

$$x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$\boldsymbol{x + \frac{1}{x} = 6} \qquad \ldots (3)$$

(ii) $x - \frac{1}{x} = ?$

Solution:

Eq. (1) − (2) =>

$$x - \frac{1}{x} = \left(3 + \sqrt{8}\right) - \left(3 - \gamma_8^{-}\right)$$

$$= 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$\boldsymbol{x - \frac{1}{x} = 2\sqrt{8}} \qquad \ldots \qquad (4)$$

(iii) $x^2 + \frac{1}{x^2} = ?$

Solution:

$$x + \frac{1}{x} = 6$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$\Rightarrow x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} = 36$$

$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 36$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 36 - 2$$

$$\Rightarrow \boldsymbol{x^2 + \frac{1}{x^2} = 34} \qquad \ldots \qquad (5)$$

(iv) $x^2 - \frac{1}{x^2} = ?$

Solution:

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Put values from eq. (3) and (4):

$$\Rightarrow x^2 - \frac{1}{x^2} = (6)\left(2\sqrt{8}\right)$$

$$\Rightarrow \boldsymbol{x^2 - \frac{1}{x^2} = 12\sqrt{8}}$$

(v) $x^4 + \frac{1}{x^4} = ?$

Solution:

eq. (5)⇒

$$x^2+\frac{1}{x^2}=34$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=(34)^2$$

$$\Rightarrow\ x^4+2x^2\cdot\frac{1}{x^2}+\frac{1}{x^4}=1156$$

$$\Rightarrow\ x^4+2+\frac{1}{x^4}=1156$$

$$\Rightarrow\ x^4+\frac{1}{x^4}=1156\ -2$$

$$\boldsymbol{x^4+\frac{1}{x^4}=1154}$$

(vi) $\left(x-\frac{1}{x}\right)^2=?$

Solution:

eq. (4)⇒

$$x-\frac{1}{x}=2\sqrt{8}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=\left(2\sqrt{8}\right)^2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=\ 4\times 8$$

$$\Rightarrow \boldsymbol{\left(x-\frac{1}{x}\right)^2=32}$$

Question No.4: Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$

Solution:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$$

$$\Rightarrow \frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = p+q\sqrt{2}$$

$$\Rightarrow \frac{\left(8-3\sqrt{2}\right)\left(4-3\sqrt{2}\right)}{\left(4+3\sqrt{2}\right)\left(4-3\sqrt{2}\right)} = p+q\sqrt{2}$$

$$\Rightarrow \frac{32-24\sqrt{2}-12\sqrt{2}+9\times 2}{(4)^2-\left(3\sqrt{2}\right)^2} = p+q\sqrt{2}$$

$$\Rightarrow \frac{32-36\sqrt{2}+18}{16-9\times 2} = p+q\sqrt{2}$$

$$\Rightarrow \frac{50-36\sqrt{2}}{16-18} = p+q\sqrt{2}$$

$$\Rightarrow \frac{50-36\sqrt{2}}{-2} = p+q\sqrt{2}$$

$$\Rightarrow \frac{50}{-2} - \frac{36\sqrt{2}}{-2} = p+q\sqrt{2}$$

$$\Rightarrow -25+18\sqrt{2} = p+q\sqrt{2}$$

$\Rightarrow$ $\boldsymbol{p=-25}$ and $\boldsymbol{q=18}$

Question No.5: Simplify the following:

(i) $\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

Solution:

$$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{(5)^{2\times\frac{3}{2}}\times(3)^{5\times\frac{3}{5}}}{(2)^{4\times\frac{5}{4}}\times(2)^{3\times\frac{4}{3}}}$$

$$= \frac{(5)^3\times(3)^3}{(2)^5\times(2)^4}$$

$$= \frac{125\times27}{(2)^{5+4}}$$

$$= \frac{125\times27}{(2)^9}$$

$$= \frac{\mathbf{3375}}{\mathbf{512}}$$

(ii) $\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216\left(3^{2x-1}\right)}$

$$= \frac{54\times\left[\left(3^3\right)^{2x}\right]^{\frac{1}{3}}}{\left(3^2\right)^{x+1}+216(3)^{2x-1}}$$

$$= \frac{54\times(3)^{6x\times\frac{1}{3}}}{(3)^{2x+2}+216(3)^{2x-1}}$$

$$= \frac{54\times(3)^{2x}}{(3)^{2x}\cdot(3)^2+216(3)^{2x}\cdot(3)^{-1}}$$

$$= \frac{54\times(3)^{2x}}{(3)^{2x}\left[(3)^2+216\cdot(3)^{-1}\right]}$$

$$= \frac{54}{\frac{9}{1}+\frac{216}{3}}$$

$$= \frac{54}{\frac{27+216}{3}}$$

$$= \frac{54 \times 3}{243}$$

$$= \frac{54}{81}$$

$$= \frac{\mathbf{3}}{\mathbf{4}}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution:

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{004}{100}\right)^{-\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^{3 \times \frac{2}{3}} \times (5)^{2 \times \frac{1}{2}}}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^{3 \times \frac{2}{3}} \times 5}{(25)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^2 \times 5}{\left(5^2\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^2 \times 5}{(5)^{2 \times \frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^2 \times 5}{(5)^3}}$$

$$= \sqrt{\frac{(6)^2}{(5)^{3-1}}}$$

$$= \sqrt{\frac{(6)^2}{(5)^2}}$$

$$= \mathbf{\frac{6}{5}}$$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \left[\left(a^{\frac{1}{3}}\right)^2 - a^{\frac{1}{3}} b^{\frac{2}{3}} + \left(b^{\frac{2}{3}}\right)^2\right]$$

$$= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3$$

$$= (a)^{\frac{1}{3} \times 3} + (b)^{\frac{2}{3} \times 3}$$

$$= \boldsymbol{a + b^2}$$